

# Electromagnetic Processes In Strongly Magnetized Plasma

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May 28, 2008

International Seminar “*Quarks-2008*”, Sergiev Posad, Moscow Region,  
May 23-29, 2008

In collaboration with Dmitry Rumyantsev

D. A. Rumyantsev, M. V. Chistyakov, JETP 101 (2005) 635  
D. A. Rumyantsev, M. V. Chistyakov, hep-ph/0609192

# Outline

- 1 Introduction
- 2 Photon dispersion properties
- 3 Transfer equation
  - Diffusion coefficient
  - Scattering coefficient
  - Absorption coefficient
- 4 Conclusion

## Introduction

Critical value of magnetic field

$$B_e = \frac{m_e^2}{e} \simeq 4.41 \cdot 10^{13} \text{ G.}$$

Magnetars: SGR, AXP.

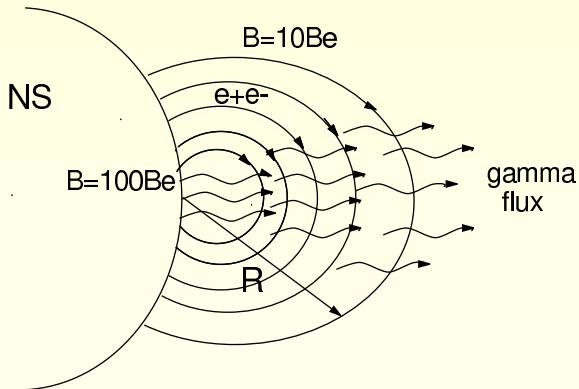
$$B \sim 10^{14} \div 10^{15} \text{ G.}$$

Strongly magnetized plasma

$$\sqrt{eB} \gg m_e, T, \mu$$

$$c = \hbar = k = 1$$

## Introduction



*C. Tompson, R.C. Duncan, MNRAS, 1995*

# Introduction

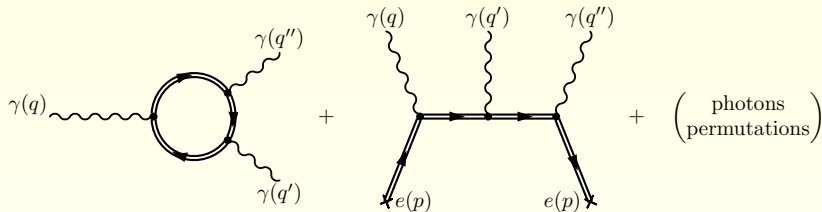
## The main processes

- Compton scattering

# Introduction

## The main processes

- Compton scattering
- Photon splitting and merging



# Introduction

## The main goal

*To demonstrate that the self-consistent accounting of strong magnetic field and dense plasma influence is necessary for the correct description of radiation transfer.*

## Photon dispersion properties...

Are defined by the eigenvectors and eigenvalues of the polarisation operator

$$\mathcal{P}_{\mu\nu}(q)\varepsilon_\nu^{(\lambda)}(q) = \mathcal{P}^{(\lambda)}(q)\varepsilon_\mu^{(\lambda)}(q).$$

Dispersion equations

$$q^2 - \mathcal{P}^\lambda(q) = 0.$$

Renormalization

$$\varepsilon_\mu^{(\lambda)} \rightarrow \varepsilon_\mu^{(\lambda)} \sqrt{Z_2}, \quad Z_\lambda^{-1} = 1 - \frac{\partial \mathcal{P}^{(\lambda)}(q)}{\partial \omega^2}.$$



## Photon dispersion properties...

Polarisation vectors in the case  $\mu = 0$

$$\varepsilon_{\mu}^{(1)} = \frac{(\varphi q)_{\mu}}{\sqrt{q_{\perp}^2}}, \quad \varepsilon_{\mu}^{(2)} = \frac{(\tilde{\varphi} q)_{\mu}}{\sqrt{q_{\parallel}^2}}, \quad \varepsilon_{\mu}^{(3)} = \frac{q_{\parallel}^2(\Lambda q)_{\mu} - q_{\perp}^2(\tilde{\Lambda} q)_{\mu}}{\sqrt{-q^2 q_{\parallel}^2 q_{\perp}^2}}$$

$\varepsilon_{\mu}^{(3)}$  - unphysical mode!

Polarisation vectors in the case  $\mu \neq 0$

$$\varepsilon_{\mu}^{(1)}(q) \simeq d_1 \varepsilon_{\mu}^{(1)} + id_2 \varepsilon_{\mu}^{(3)}, \quad \varepsilon_{\mu}^{(3)}(q) \simeq d_3 \varepsilon_{\mu}^{(1)} + id_4 \varepsilon_{\mu}^{(3)},$$

$$\varepsilon_{\mu}^{(2)}(q) \simeq \frac{(\tilde{\varphi} q)_{\mu}}{\sqrt{q_{\parallel}^2}}.$$

$$\varphi_{\alpha\beta} = F_{\alpha\beta}/B, \quad \tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}, \quad (a\varphi b) = a_{\alpha}\varphi_{\alpha\beta}b_{\beta}, \quad (ab)_{\parallel} = (a\tilde{\Lambda}b) = (a\tilde{\varphi}\tilde{\varphi}b), \\ (ab)_{\perp} = (a\Lambda b) = (a\varphi\varphi b).$$

## Photon dispersion properties...

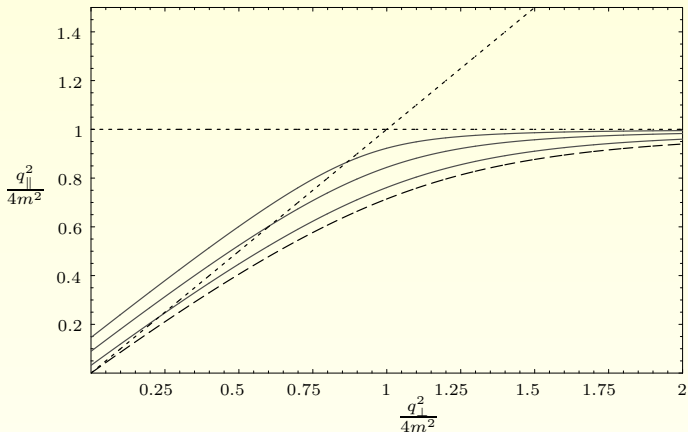


Figure:  $B = 200B_e$ ,  $T = 0.25; 0.5; 1MeV$ ,  $\theta = \pi/2$

# Photon dispersion properties...

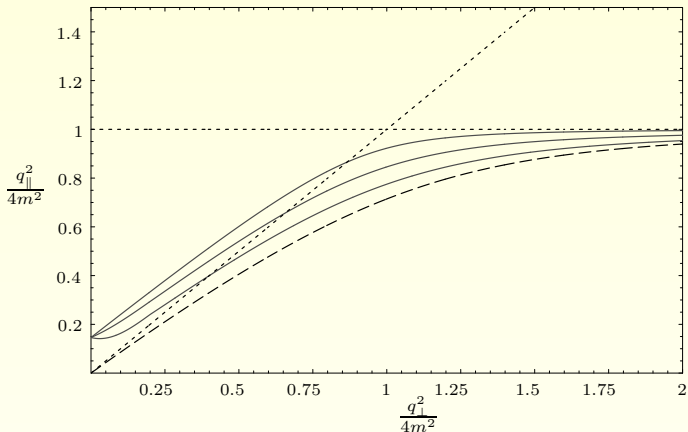


Figure:  $B = 200B_e$ ,  $T = 1MeV$ ,  $\theta = \pi/12; \pi/6; \pi/2$

# Photon dispersion properties...

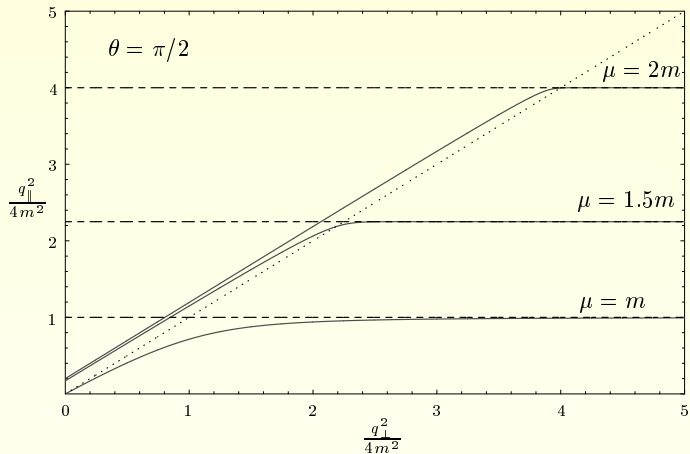


Figure:  $B = 200B_e$

## Photon dispersion properties...

### Possible channels

- mode 1 (extraordinary photon)

$$\gamma_1 e^\pm \rightarrow \gamma_1 e^\pm, \gamma_1 e^\pm \rightarrow \gamma_2 e^\pm, \gamma_1 \rightarrow \gamma_1 \gamma_2, \\ \gamma_1 \rightarrow \gamma_2 \gamma_2, \gamma_1 \gamma_2 \rightarrow \gamma_1, \gamma_1 \gamma_1 \rightarrow \gamma_2;$$

- mode 2 (ordinary photon)

$$\gamma_2 e^\pm \rightarrow \gamma_2 e^\pm, \gamma_2 e^\pm \rightarrow \gamma_1 e^\pm, \gamma_2 \rightarrow \gamma_1 \gamma_1, \\ \gamma_2 \gamma_2 \rightarrow \gamma_1, \gamma_2 \gamma_1 \rightarrow \gamma_1.$$

## Photon dispersion properties...

$$\sigma_{1 \rightarrow 1} = \frac{3}{4} \sigma_T \left( \frac{B_e}{B} \right)^2 \frac{\omega^2}{m^2},$$

$$\sigma_{1 \rightarrow 2} = \frac{1}{4} \sigma_T \left( \frac{B_e}{B} \right)^2 \frac{\omega^2}{m^2},$$

$$\sigma_{2 \rightarrow 1} = \frac{3}{4} \sigma_T \left( \frac{B_e}{B} \right)^2 \frac{(\omega - \omega_{pl})^2}{m^2} u^2 \Theta(\omega - \omega_{pl}),$$

$$\sigma_{2 \rightarrow 2} = \sigma_T \left\{ (1 - u^2) + \left( \frac{B_e}{B} \right)^2 \frac{(\omega - \omega_{pl})^2}{m^2} u^2 \right\} \Theta(\omega - \omega_{pl}),$$

## Transfer Equation...

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 D_1 \frac{dn_1}{dr} \right) + K_1(\bar{n} - n_1) + S_{12}(n_2 - n_1) = 0,$$

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 D_2 \frac{dn_2}{dr} \right) + K_2(\bar{n} - n_2) + S_{21}(n_1 - n_2) = 0,$$

$$\bar{n} = \frac{\omega^3}{2\pi^2} f_\omega, \quad f_\omega = [\exp(\omega/T) - 1]^{-1}$$

# Transfer Equation...

Diffusion, absorption and scattering coefficients

$$D_{\lambda} = \int \frac{d\Omega}{4\pi} \ell_{\lambda}(\theta, r) \cos^2 \theta,$$

$$K_{\lambda} = \int \frac{d\Omega}{4\pi} [W_{\lambda \rightarrow \lambda' \lambda''}(\theta, r) + W_{\lambda \lambda' \rightarrow \lambda''}(\theta, r)],$$

$$S_{\lambda \lambda'} = \int \frac{d\Omega}{4\pi} W_{\lambda \rightarrow \lambda'}(\theta, r),$$

$$\ell_{\lambda} = \left[ \sum_{\lambda'=1}^2 W_{\lambda \rightarrow \lambda'} + \sum_{\lambda', \lambda''=1}^2 (W_{\lambda \rightarrow \lambda' \lambda''} + W_{\lambda \lambda' \rightarrow \lambda''}) \right]^{-1}$$



## Transfer Equation...

- photon scattering

$$W_{\lambda \rightarrow \lambda'} = \frac{eB}{16(2\pi)^4 \omega_\lambda} \int |\mathcal{M}_{\lambda \rightarrow \lambda'}|^2 Z_\lambda Z_{\lambda'} \times \\
 \times f_E (1 - f_{E'}) (1 + f_{\omega'}) \delta(\omega_\lambda(\mathbf{k}) + E - \omega_{\lambda'}(\mathbf{k}') - E') \frac{dp_z d^3 k'}{EE' \omega_{\lambda'}};$$

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- photon splitting

$$W_{\lambda \rightarrow \lambda' \lambda''} = \frac{1 - (1/2)\delta_{\lambda' \lambda''}}{32\pi^2 \omega} \int |\mathcal{M}_{\lambda \lambda' \lambda''}|^2 Z_\lambda Z_{\lambda'} Z_{\lambda''} \times \\
 \times (1 + f_{\omega'}) (1 + f_{\omega''}) \delta(\omega_\lambda(\mathbf{k}) - \omega_{\lambda'}(\mathbf{k} - \mathbf{k}'') - \omega_{\lambda''}(\mathbf{k}'')) \frac{d^3 k''}{\omega_{\lambda'} \omega_{\lambda''}},$$

## Transfer Equation...

- photon merging

$$W_{\lambda\lambda' \rightarrow \lambda''} = \frac{1}{32\pi^2\omega} \int |\mathcal{M}_{\lambda\lambda'\lambda''}|^2 Z_\lambda Z_{\lambda'} Z_{\lambda''} \times \\
 \times f_{\omega'} (1 + f_{\omega''}) \delta(\omega_\lambda(\mathbf{k}) + \omega_{\lambda'}(\mathbf{k}') - \omega_{\lambda''}(\mathbf{k} + \mathbf{k}')) \frac{d^3k'}{\omega_{\lambda'}\omega_{\lambda''}}.$$

## Difusion coefficient...

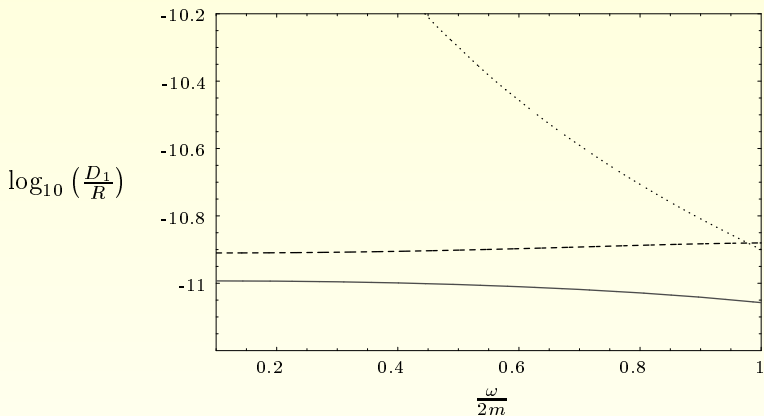


Figure:

$$B = 100B_e; 60B_e, \quad T = 1MeV; 0.5MeV, \quad \ell_1^{-1} = n_e \sigma_T (B_e \omega / Bm)^2$$

## Difusion coefficient...

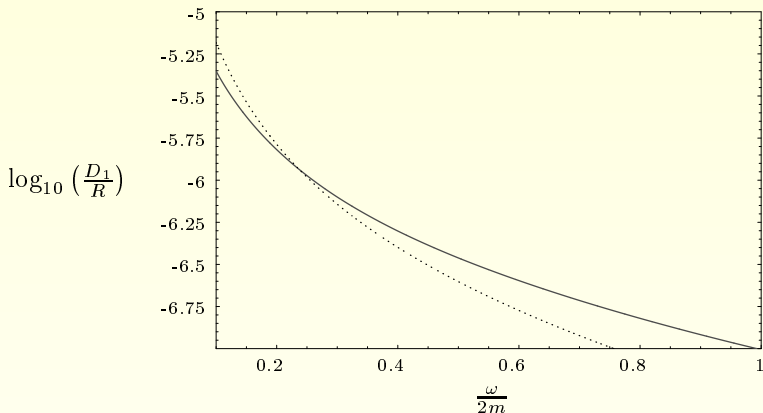


Figure:  $B = 10B_e$ ,  $T = 0.05MeV$

$$\ell_1^{-1} = n_e \sigma_T (B_e \omega / B m)^2 + (\alpha^3 \sin^6 \theta / 2160 \pi^2) (\omega / m)^5 m$$

## Difusion coefficient...

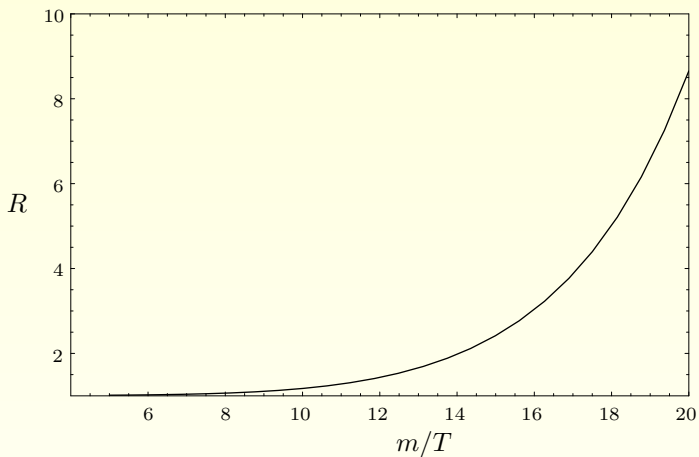


Figure:  $B = 200B_e$ ,

$$R = D_1^0 / D_1^{\text{splitting}}$$

## Scattering coefficient ...

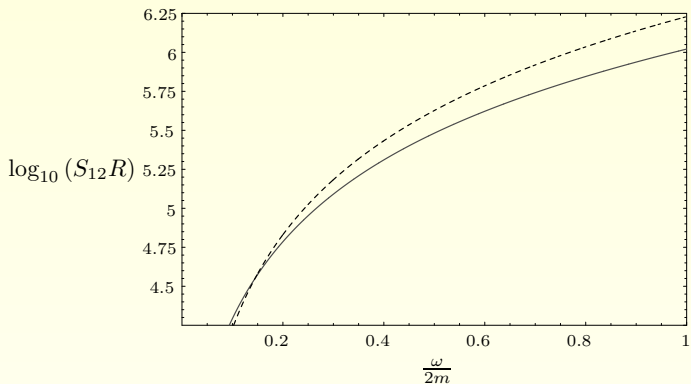


Figure:  $B = 10B_e$ ,  $T = 0.05MeV$ ,  $\ell_1^{-1} = n_e \sigma_T (B_e \omega / Bm)^2$

## Absorption coefficient ...

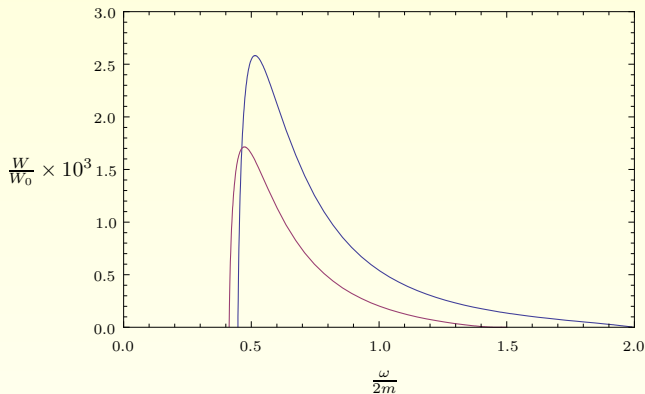


Figure:  $\gamma_2 \rightarrow \gamma_1 \gamma_1$ ,  $B = 200 B_e$ ,  $\mu = 1.5m; 2m$ ,  
 $W_0 = (\alpha/\pi)^3 m \simeq 3.25 \cdot 10^2 \text{ cm}^{-1}$ .



# Absorption coefficient ...

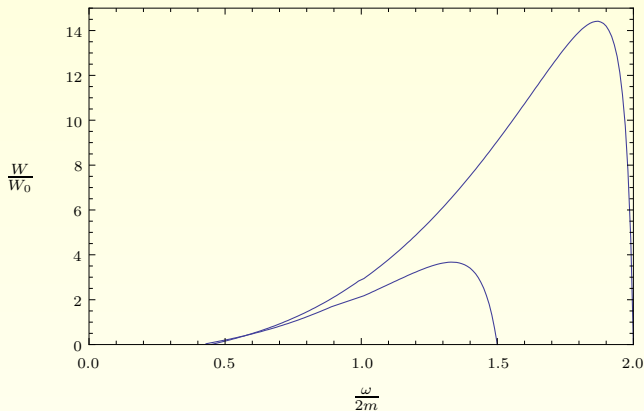


Figure:  $\gamma_2 e^\pm \rightarrow \gamma_1 e^\pm$ ,  $B = 200 B_e$ ,  $\mu = 1.5m; 2m$ ,  
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- Photon dispersion and radiative correction in strongly magnetized plasma could essentially influence on the radiation transfer process.

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- Photon dispersion and radiative correction in strongly magnetized plasma could essentially influence on the radiation transfer process.
- In charge symmetric plasma ( $\mu = 0$ ) it is necessary to take into account the processes of photon splitting and photon merging.
- In strongly degenerate plasma the influence of the photon splitting and photon merging processes on radiation transfer is negligibly small.