Compton scattering in strongly magnetized plasma

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Contents

Introduction

Critical value of magnetic field

$$B_e = \frac{m_e^2}{e} \simeq 4.41 \times 10^{13} \text{ G}, \qquad c = \hbar = k = 1.$$

Magnetars
SGR – soft gamma repeaters,
anomalous X-ray pulsars (AXP).

$B \sim 10^{15}$ G.
SGR 1806-20 – $B \sim 7 \times 10^{15}$ G (Israel et al. astro-ph/0505255).

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Historical review

Melrose, Parle 1983 - modification of the scattering amplitude

Harding et al. 1986, 2000 - cross section in the limit $\omega \ll 2m$

Elmfors et al. 1998 - Compton scattering could compete with the photon splitting

The astrophysical application

Duncan, Thompson 1995 - Compton scattering in the limit $T \ll m$, is suppressed in comparison with the photon splitting channel $\gamma_1 \to \gamma_2 \gamma_2$

This is correct only at $T \ll 25$ keV $(\mu = 0)$

The limit of strongly magnetized plasma

$$eB \gg T^2, \mu^2, \omega^2, E^2$$

The region is below cyclotron resonance: $eB \gg (pk)$

Photon dispersion properties in a strongly magnetized and charge-symmetric plasma $(\mu = 0)$

The eigenvalues of the photon polarization operator in plasma could be presented in the following form

$$\begin{aligned}
\mathcal{P}^{(1)}(q) &\simeq -\frac{\alpha}{3\pi}\, q_\perp^2 - q^2\,\Lambda(B), \\
\mathcal{P}^{(2)}(q) &\simeq -\frac{2eB\alpha}{\pi}\left[H\left(\frac{4m^2}{q_\parallel^2}\right) + \mathcal{J}(q_\parallel)\right] - q^2\,\Lambda(B), \\
\mathcal{P}^{(3)}(q) &\simeq -\,q^2\,\Lambda(B),
\end{aligned}$$

where $\Lambda(B) = \frac{\alpha}{3\pi}\left[1.792 - \ln(B/B_e)\right]$,

$$\mathcal{J}(q_\parallel) = 4q_\parallel^2 m^2 \int \frac{dp_z}{E}\,\frac{f_E}{(q_\parallel^2)^2 - 4(pq)_\parallel^2}\,, \qquad E = \sqrt{p_z^2 + m^2},$$

$f_E = [\exp(E/T) + 1]^{-1}$ is the electron distribution function,

$$H(z) = \frac{z}{\sqrt{z-1}}\arctan\frac{1}{\sqrt{z-1}} - 1,\ z \geq 1.$$

Photon dispersion ...

Dispersion equation

$$q^2 - \mathcal{P}^{(\lambda)}(q) = 0 \qquad (\lambda = 1, 2, 3).$$

Polarization vectors $(\lambda = 1, 2)$

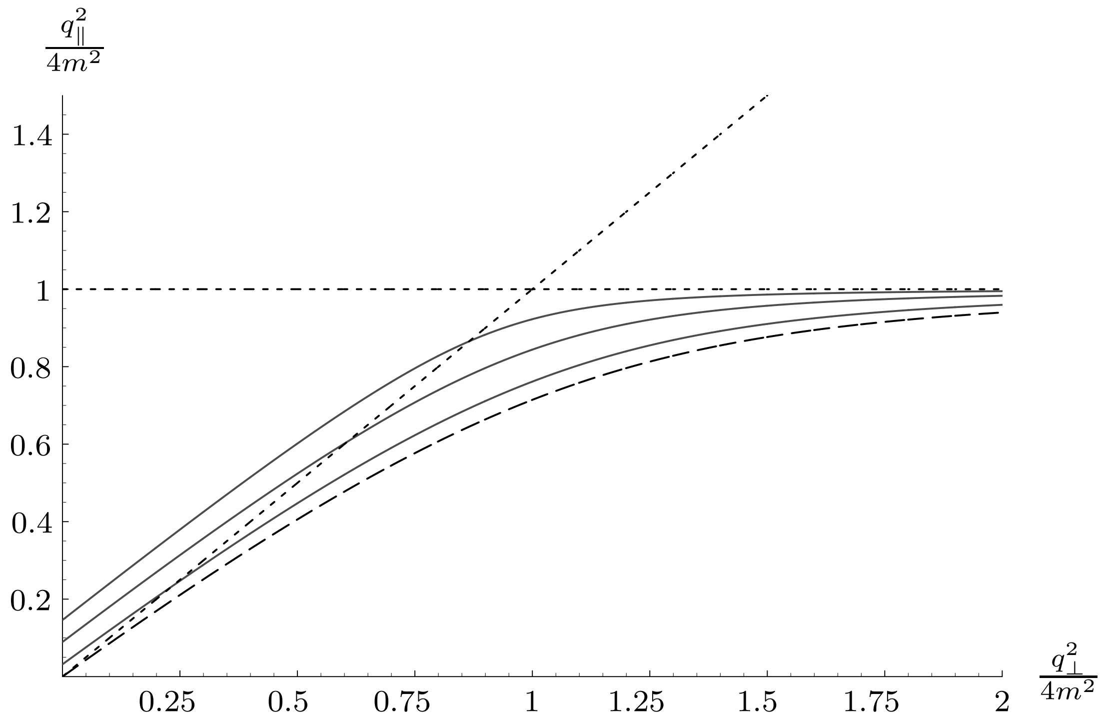
$$\varepsilon_\alpha^{(1)}(q) \simeq \frac{(\varphi q)_\alpha}{\sqrt{(q \varphi \varphi q)}}, \qquad \varepsilon_\alpha^{(2)}(q) \simeq \frac{(\tilde{\varphi} q)_\alpha}{\sqrt{(q \tilde{\varphi} \tilde{\varphi} q)}}.$$

Renormalization of the mode 2 of the photon wave function

$$\varepsilon_\alpha^{(2)} \to \varepsilon_\alpha^{(2)} \sqrt{Z_2}, \quad Z_2^{-1} = 1 - \frac{\partial \mathcal{P}^{(2)}}{\partial \omega^2}.$$

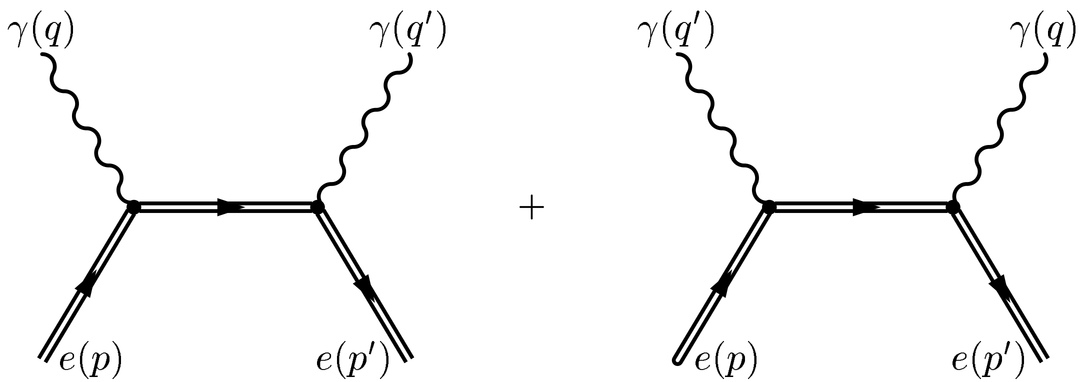
The four-vectors with indices $\perp$ and $\parallel$ belong to the Euclidean $\{1, 2\}$-subspace and the Minkowski $\{0, 3\}$-subspace correspondingly in the frame were the magnetic field is directed along the third axis; $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}$ are the dimensionless field tensor and dual field tensor correspondingly. The tensors $\Lambda_{\alpha\beta} = (\varphi\varphi)_{\alpha\beta}$, and $\widetilde{\Lambda}_{\alpha\beta} = (\tilde{\varphi}\tilde{\varphi})_{\alpha\beta}$ related by $\widetilde{\Lambda}_{\alpha\beta} - \Lambda_{\alpha\beta} = g_{\alpha\beta} = diag(1, -1, -1, -1)$ are introduced.

Photon dispersion ...



Photon dispersion laws in the strong magnetic field $B/B_e = 200$ and neutral plasma at the temperature: $T = 1$ MeV (upper solid curve), $T = 0.5$ MeV (middle solid curve), $T = 0.25$ MeV (lower solid curve). The photon dispersion without plasma is depicted by the dashed line. The dotted line corresponds to the vacuum dispersion law, $q_\parallel^2 - q_\perp^2 = 0$.

The Feynman diagrams for the Compton process in magnetic field.



Amplitudes of the photon scattering

$$\mathcal{M}_{1\to 1} = -\frac{8\pi\alpha m}{eB}\,\frac{(q\varphi q')(q\tilde{\varphi} q')}{\sqrt{q_\perp^2 q_\perp'^2 (-Q_\parallel^2)}},$$

$$\mathcal{M}_{1\to 2} = -\frac{8\pi\alpha m}{eB}\,\frac{(q\Lambda q')(q'\tilde{\Lambda} Q)}{\sqrt{q_\perp^2 q_\parallel'^2 (-Q_\parallel^2)}},$$

$$\mathcal{M}_{2\to 1} = \frac{8\pi\alpha m}{eB}\,\frac{(q\Lambda q')(q\tilde{\Lambda} Q)}{\sqrt{q_\parallel^2 q_\perp'^2 (-Q_\parallel^2)}},$$

$$\mathcal{M}_{2\to 2} = 16 i\pi\alpha m\,\frac{\sqrt{q_\parallel^2 q_\parallel'^2}\,\sqrt{(-Q_\parallel^2)}\,\varkappa}{(q\tilde{\Lambda} q')^2 - \varkappa^2 (q\tilde{\varphi} q')^2},$$

where $\varkappa = \sqrt{1 - 4m^2/Q_\parallel^2}$, $Q_\parallel^2 = (q - q')_\parallel^2 < 0$.

The photon scattering absorption coefficient

The general expression for the photon scattering absorption coefficient by real electrons and positrons of the medium can be written in the following form

$$W_{\lambda e^{\pm} \to \lambda' e^{\pm}} = \frac{eB}{16(2\pi)^4 \omega_\lambda} \int | \mathcal{M}_{\lambda \to \lambda'} |^2 Z_\lambda Z_{\lambda'} \times$$

$$\times f_E \left(1 - f_{E'}\right) \left(1 + f_{\omega'}\right) \delta(\omega_\lambda(\mathbf{k}) + E - \omega_{\lambda'}(\mathbf{k}') - E') \frac{dp_z \, d^3k'}{E E' \omega_{\lambda'}},$$

where $f_{\omega'} = [\exp(\omega'/T) - 1]^{-1}$ is the photon distribution function.

The analytical expression for the photon scattering absorption coefficient in the case of a rare electron gas ($T \ll m$) can be presented as

$$W_{\lambda \to \lambda'} = W_{\lambda e^- \to \lambda' e^-} + W_{\lambda e^+ \to \lambda' e^+} = n_e \sigma_{\lambda \to \lambda'},$$

$$n_e = eB \sqrt{\frac{mT}{2\pi^3}} \, e^{-m/T}.$$

The absorption coefficient ...

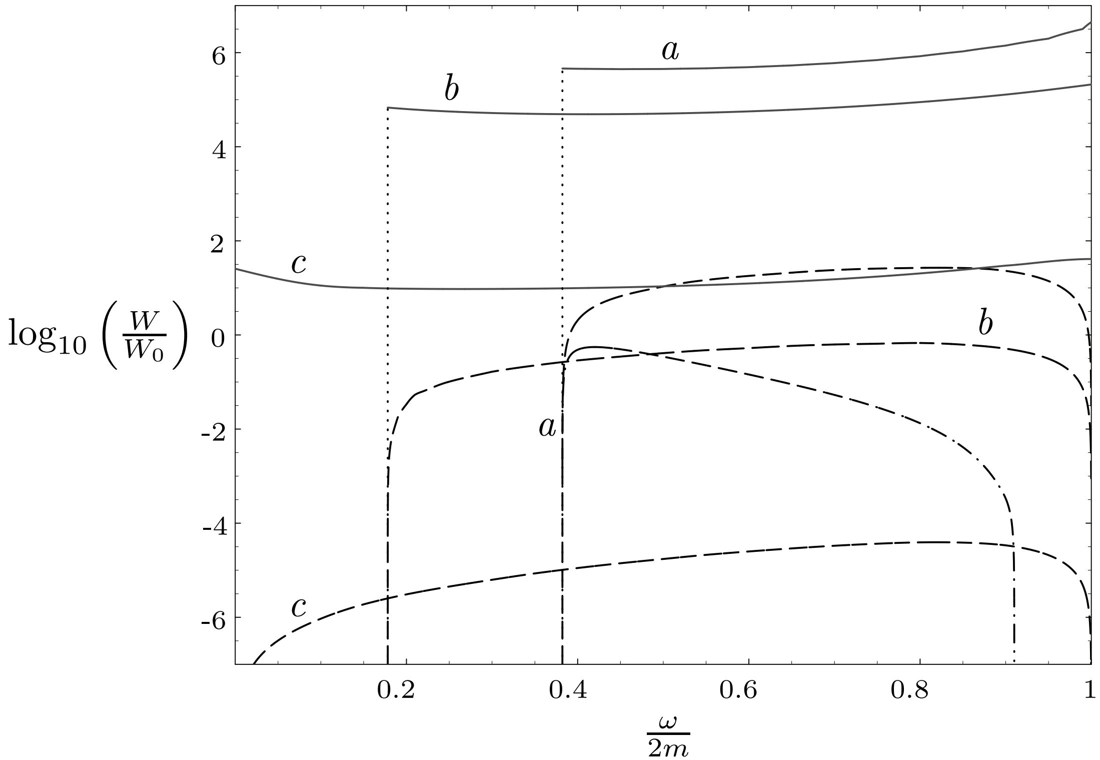
$$\sigma_{1\to 1} = \frac{\alpha^2 \pi}{(eB)^2}\, \omega^2 \left[\frac{\omega + 2m}{2(\omega + m)} + \frac{m}{\omega} \ln\left(1 + \frac{\omega}{m}\right) \right],$$

$$\sigma_{2\to 1} = \frac{\alpha^2 \pi Z_2}{(eB)^2}\, q_\perp^2 \left[\frac{\omega + 2m}{2(\omega + m)} - \frac{m}{\omega} \ln\left(1 + \frac{\omega}{m}\right) \right],$$

$$\sigma_{1\to 2} = \frac{\alpha^2 \pi (\omega + 2m)^2}{2(eB)^2 \omega (\omega + m)} \int\limits_0^{\omega^2} dq_\parallel'^2 \left(1 + \frac{2\alpha eB}{\pi} \frac{1}{q_\parallel'^2} H\left(\frac{4m^2}{q_\parallel'^2}\right)\right) \times$$
$$\times \sqrt{\frac{\omega^2 - q_\parallel'^2}{(\omega + 2m)^2 - q_\parallel'^2}},$$

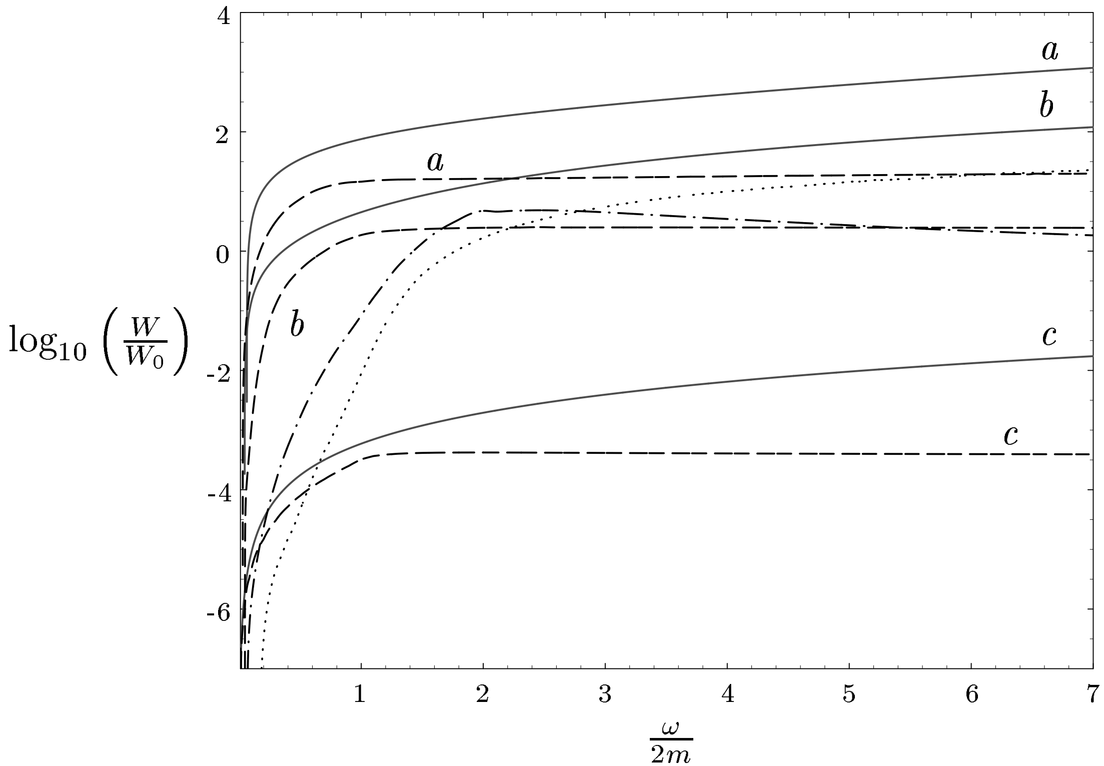
$$\sigma_{2\to 2} = \frac{16 m^2 \alpha^2 \pi (\omega + m)}{\omega^3 (\omega + 2m)^2} Z_2 \left[\frac{\omega(\omega + 2m)}{(\omega + m)(2m - \omega)} - \ln\left(1 + \frac{\omega}{m}\right) + \right.$$
$$\left. + \frac{2\omega(\omega - m)(2m + \omega)}{(\omega + m)(2m - \omega)\sqrt{4m^2 - \omega^2}} \arctan \frac{\omega}{\sqrt{4m^2 - \omega^2}} \right].$$

The absorption coefficient ...



The dependence of the absorption coefficient of photon scattering of channels $\gamma_2 e^{\pm} \to \gamma_2 e^{\pm}$ (solid line) and $\gamma_2 e^{\pm} \to \gamma_1 e^{\pm}$ (dashed line) on the energy of the initial photon in a strong magnetic field $B/B_e = 200$ and neutral ($\mu = 0$) plasma, at $T = 1MeV$ – *a*, $T = 250keV$ – *b*, $T = 50keV$ – *c*. The chain line corresponds to the probability of photon splitting, $\gamma_2 \to \gamma_1\gamma_1$, at $T = 1MeV$. Here $W_0 = (\alpha/\pi)^3\, m$.

The absorption coefficient ...



The dependence of the absorption coefficient of photon scattering of channels $\gamma_1 e^\pm \to \gamma_1 e^\pm$ (firm line) and $\gamma_1 e^\pm \to \gamma_2 e^\pm$ (dashed line) on energy of initial photon in a strong magnetic field $B/B_e = 200$ and neutral $(\mu = 0)$ plasma, at $T = 1 MeV$ – *a*, $T = 250 keV$ – *b*, $T = 50 keV$ – *c*. The dotted and chain lines correspond to the probability of photon splitting, $\gamma_1 \to \gamma_1\gamma_2$ and $\gamma_1 \to \gamma_2\gamma_2$, respectively, at $T = 50 keV$.

The absorption coefficient ...

The channels $\gamma_2 e^{\pm} \to \gamma_2 e^{\pm}$ and $\gamma_2 e^{\pm} \to \gamma_1 e^{\pm}$ are dominated over the photon splitting channel $\gamma_2 \to \gamma_1 \gamma_1$.

The absorption coefficient of scattering $\gamma_1 e^{\pm} \to \gamma_1 e^{\pm}$ is compared with photon splitting at $\omega < 2m$ and $T \leq 25$ keV

Low limit of the photon energy can be obtained from equation

$$\int_{R^*}^{R} dr W_{\lambda \to \lambda'}(r, \omega_{sc}) = 1.$$

$T \geq 16$ keV $\Longrightarrow$ the photon of the second mode can't leave the region filled by the strong magnetic field and plasma.

The result for the channel $\gamma_1 e^{\pm} \to \gamma_1 e^{\pm}$ is $\omega_{sc} \simeq 85$ keV ($T = 25$ keV).

Duncan and Thompson obtained that $\omega_{sp} \simeq 37.5$ keV in the channel $\gamma_1 \to \gamma_2 \gamma_2$.

Summary

- The process of Compton scattering, $\gamma e^{\pm} \to \gamma e^{\pm}$, in a strongly magnetized medium of an arbitrary temperature and zeroth chemical potential is considered.
- The analytical expressions for the partial cross section in the small number of density limit of the electron-positron plasma are obtained.
- The numerical estimations for partial probabilities of this process are presented by taking into account the photon dispersion in a strong magnetic field and a charge-symmetric plasma of an arbitrary temperature.
- The comparison of the scattering probability with photon splitting in a plasma of arbitrary temperatures was performed. The results show, that the photon scattering and photon splitting channels are comparable at $T \geq 25$ keV and magnetic field strength $B = 200 B_e$.
- The estimation of the low limit of photon energy, at which the optical depth is equal to one was obtained.

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