Neutrino dispersion in external magnetic field and plasma

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Outline

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- Field-induced resonance $\nu_{\tau,\mu} \longrightarrow \nu_e$ transition
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- Conclusions

Introduction

Solving the Solar Neutrino Puzzle at Sudbury

$\Rightarrow$ Pontecorvo's idea on neutrino oscillations confirmed

$\Rightarrow$ Indirect evidence of neutrino masses and mixings

$\Rightarrow$ Enthusiasm among theorists: are there more observable effects of medium influence on neutrinos?

$\Rightarrow$ Theoretical discoveries $\Rightarrow$ Theoretical closings $\Downarrow$

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Introduction

Matter influence on neutrino dispersion properties — MSW mechanism of the resonance enhancement of the neutrino oscillations in matter.

External magnetic field influence?

A natural scale for the field strength to have a significant impact on quantum processes: **the critical value**

$$B_e = m_e^2/e \approx 4.41 \times 10^{13}\ \mathrm{G}$$

$$(B_\mu = m_\mu^2/e \simeq 2 \times 10^{18}\ \mathrm{G}\,, \quad B_W = m_W^2/e \simeq 10^{24}\ \mathrm{G})$$

Let's call the fields $eB \ll m_e^2$ as **weak**, and $m_e^2 \ll eB \ll m_W^2$ as **moderate**.

Introduction

Natural laboratories of gigantic neutrino outflow + superstrong fields: cataclysmic astrophysical events of supernova explosions or coalescing neutron stars.

Remnants of such cataclysms: soft gamma-ray repeaters (SGR) and anomalous x-ray pulsars (AXP) = **magnetars**, neutron stars with magnetic fields $\sim 10^{14}$–10^{15} G.

One more natural laboratory with strong magnetic fields and large neutrino densities: the early universe between the QCD phase transition ($\sim 10^{-5}$ s) and the nucleosynthesis epoch ($\sim 10^{-2}$–10^{+2} s).

Introduction

Dispersion relation for electron neutrino in a charge-symmetric plasma with $m_e \ll T \ll m_W$ and $B \lesssim T^2$, ignoring the neutrino mass (J. C. D'Olivo e.a., 1989; P. Elmfors e.a., 1996; A. Erdas e.a., 1998):

$$\frac{E}{|\mathbf{p}|} = 1 + \frac{\sqrt{2}\, G_{\rm F}}{3} \left[-\frac{7\pi^2 T^4}{15} \left(\frac{1}{m_Z^2} + \frac{2}{m_W^2} \right) + \frac{T^2 eB}{m_W^2} \cos\phi + \right.$$

$$\left. + \frac{(eB)^2}{\pi^2 m_W^2} \ln\left(\frac{T}{m_e}\right) \sin^2\phi \right],$$

$\mathbf{p}$ – the neutrino momentum, ϕ – the angle between $\mathbf{B}$ and $\mathbf{p}$.

The B-field induced pure vacuum modification of the neutrino dispersion relation was assumed to be negligible in these papers.

Introduction

However, recent calculation by E. Elizalde, E. Ferrer, V. de la Incera, 2002; 2004, gave an absolutely different result:

$$\frac{\Delta E}{|\mathbf{p}|} = \sqrt{2}\, G_{\mathrm{F}}\, \frac{eB}{8\pi^2}\, \sin^2\phi\, \mathrm{e}^{-p^2 \sin^2\phi/(2eB)} \,,$$

This would be the dominant B-field induced contribution

$\Rightarrow$ important consequences for neutrino physics in media.

The B-field contribution into the neutrino dispersion relation was dominating or negligible?

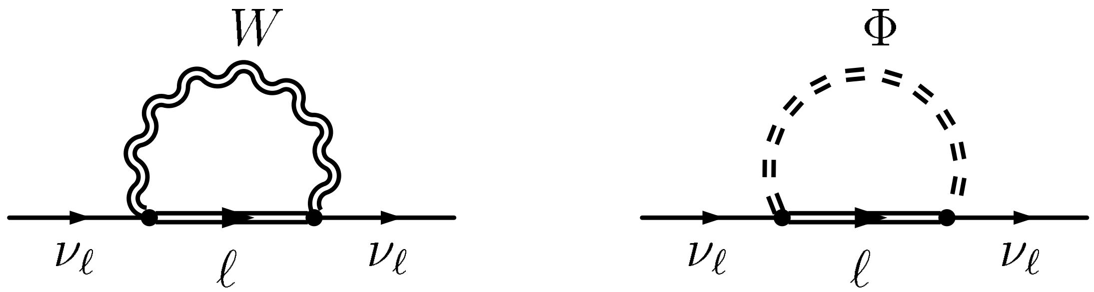
An independent calculation **was strongly urged**.

The neutrino self-energy operator $\Sigma(p)$

The operator $\Sigma(p)$ is defined via the amplitude for the neutrino forward scattering on vacuum fluctuations, $\nu \longrightarrow \nu$

$$\mathcal{M}(\nu \to \nu) = -\bar{\nu}(p)\,\Sigma(p)\,\nu(p)$$

The Feynman diagrams in the Feynman gauge:



The contribution of the diagram with scalar is suppressed by the factor $(m_\ell/m_W)^2$, however, it is essential in some cases (hep-ph/0605114).

The neutrino self-energy operator $\Sigma(p)$

The calculation techniques for quantum processes in external electromagnetic fields based on **exact propagators in the field**, started from the classical paper by J. Schwinger (1951) and was developed by A. Nikishov, V. Ritus, A. Shabad, V. Skobelev et al.

For a recent review see e.g. A. K., N. M., *Electroweak Processes in External Electromagnetic Fields* (Springer-Verlag, New York, 2003).

The neutrino self-energy operator $\Sigma(p)$

A general covariant structure of the $\Sigma(p)$ operator in an external magnetic field:

$$\Sigma(p) = \left[\mathcal{A}_L\,(p\gamma) + \mathcal{B}_L\,e^2(p\tilde{F}\tilde{F}\gamma) + \mathcal{C}_L\,e\,\left(p\tilde{F}\gamma\right)\right]\,L +$$

$$+\left[\mathcal{A}_R\,(p\gamma) + \mathcal{B}_R\,e^2(p\tilde{F}\tilde{F}\gamma) + \mathcal{C}_R\,e\,\left(p\tilde{F}\gamma\right)\right]\,R +$$

$$+\,m_\nu\,\left[\mathcal{K}_1 + \mathrm{i}\,\mathcal{K}_2\,e(\gamma F\gamma)\right]\,,$$

F is the external field tensor, and $\tilde{F}$ is its dual,
$L, R = \frac{1}{2}(1 \mp \gamma_5)$ are the left-handed and right-handed projection operators.
The Lorentz indices of four-vectors and tensors within parentheses are contracted consecutively, e.g.
$(p\tilde{F}\gamma) = p^\alpha \tilde{F}_{\alpha\beta}\gamma^\beta$.

The neutrino self-energy operator $\Sigma(p)$

$$\Sigma(p) = \left[\mathcal{A}_L \, (p\gamma) + \mathcal{B}_L \, e^2 (p\tilde{F}\tilde{F}\gamma) + \mathcal{C}_L \, e \left(p\tilde{F}\gamma\right)\right] \, L +$$

$$+ \left[\mathcal{A}_R \, (p\gamma) + \mathcal{B}_R \, e^2 (p\tilde{F}\tilde{F}\gamma) + \mathcal{C}_R \, e \left(p\tilde{F}\gamma\right)\right] \, R +$$

$$+ m_\nu \, \left[\mathcal{K}_1 + \mathrm{i}\, \mathcal{K}_2 \, e (\gamma F \gamma)\right] \, ,$$

The coefficients $\mathcal{A}_L$, $\mathcal{A}_R$ and $\mathcal{K}_1$, being ultraviolet divergent, are absorbed by the neutrino wave-function and mass renormalization.

The coefficients $\mathcal{K}_2$ is suppressed by the factor $(m_\ell/m_W)^2$, while $\mathcal{B}_R$, $\mathcal{C}_R$ by the factor $(m_\nu/m_W)^2$.

Thus, the coefficients $\mathcal{B}_L$, $\mathcal{C}_L$ are of the most interest.

The neutrino self-energy operator $\Sigma(p)$

Authors	Field strength	$\mathcal{B}_L \times \dfrac{\sqrt{2}\,\pi^2}{G_{\rm F}}$	$\mathcal{C}_L \times \dfrac{\sqrt{2}\,\pi^2}{G_{\rm F}}$
McKeon (1981)	Moderate	0	$+3$
Borisov et al. (1985)	Arbitrary	—	$+\dfrac{3}{4}$
Erdas, Feldman (1990)	Moderate	$-\dfrac{1}{3m_W^2}\left(\ln\dfrac{m_W^2}{m_\ell^2}+\dfrac{3}{4}\right)$	0
Elizalde et al. (2002)	Moderate	$+\dfrac{1}{2eB}$	$-\dfrac{1}{2}$
Elizalde et al. (2004)	Moderate	$+\dfrac{1}{4eB}\,\mathrm{e}^{-p_\perp^2/(2eB)}$	$-\dfrac{1}{4}\,\mathrm{e}^{-p_\perp^2/(2eB)}$
Our result (2005)	Weak	$-\dfrac{1}{3m_W^2}\left(\ln\dfrac{m_W^2}{m_\ell^2}+\dfrac{3}{4}\right)$	$+\dfrac{3}{4}$
Our result (2005)	Moderate	$-\dfrac{1}{3m_W^2}\left(\ln\dfrac{m_W^2}{eB}+2.54\right)$	$+\dfrac{3}{4}$

Neutrino energy in a magnetic field

Solving the equation for the neutrino dispersion in a magnetic field (for $m_\nu = 0$)

$$\det \left|(p\gamma) - \mathcal{B}_L \, e^2 (p\tilde{F}\tilde{F}\gamma) \, L - \mathcal{C}_L \, e (p\tilde{F}\gamma) \, L\right| = 0 \, ,$$

one obtains for the neutrino energy in the field

$$\frac{E}{|\mathbf{p}|} = 1 + \left(\mathcal{B}_L + \frac{{\mathcal{C}_L}^2}{2} \right) (eB)^2 \, \sin^2 \phi \, .$$

The main contribution comes from the $\mathcal{B}_L$ coefficient, because the value $\mathcal{C}_L^2/\mathcal{B}_L \sim G_\mathrm{F} \, m_W^2$ appears to be of the order of the fine-structure constant $\alpha \simeq 1/137$, thus leading us beyond the the one-loop approximation.

Neutrino energy in a magnetic field

Our results strongly disagree with those by E. Elizalde e.a., 2002; 2004. We confirm the result by J. C. D'Olivo e.a., 1989; P. Elmfors e.a., 1996, that the pure magnetic field contribution into the neutrino energy does not exceed the plasma contribution.

For relatively weak field $eB \ll m_e^2$ we find the following pure-field correction to the electron neutrino energy in a magnetic field and plasma:

$$\frac{E}{|\mathbf{p}|} = 1 + \frac{\sqrt{2}\,G_{\mathrm{F}}}{3}\left[-\frac{7\pi^2 T^4}{15}\left(\frac{1}{m_Z^2} + \frac{2}{m_W^2}\right) + \frac{T^2 eB}{m_W^2}\cos\phi \right.$$

$$\left. + \frac{(eB)^2}{\pi^2 m_W^2}\sin^2\phi\left(\ln\frac{T}{m_e} - \ln\frac{m_W}{m_e} - \frac{3}{8}\right)\right].$$

Field-induced resonance $\nu_{\tau,\mu} \to \nu_e$ transition

A problem of the stalled **shock wave revival** in the **supernova** (Wilson,1985; Bethe,1985):

couldn't it be solved by the **resonance enhancement of the neutrino oscillations** $\nu_{\mu,\tau} \to \nu_e$ in a strong magnetic field inside the exploding supernova (Bisnovatyi-Kogan, 1970; Balbus, Hawley, 1998; Ardeljan et al., 2004)?

For the case $m_e^2 \ll eB \ll m_\ell^2 \ll m_W^2$, when $\Delta E_{MF} = E_{\nu_\ell} - E_{\nu_e}(\ell = \mu, \tau)$, the resonance condition for the $\nu_\ell \to \nu_e$ oscillation is

$$\frac{\Delta m_\nu^2}{2E} \cos 2\theta + \frac{G_F\,(eB)^2}{3\sqrt{2}\pi^2}\,\frac{E\,\sin^2\phi}{m_W^2}\left(\ln\frac{m_\ell^2}{eB} + 1.8\right) - \frac{\sqrt{2}\,G_F\,\rho\,Y_e}{m_N} = 0\,,$$

and the sign of the field-induced term is favorable!

Field-induced resonance $\nu_{\tau,\mu} \to \nu_e$ *transition*

The magnetic field strength providing the resonance transition $\nu_{\tau,\mu} \to \nu_e$, evaluated from the equation

$$B_{17}^2 \, (1 - 0.10 \times \ln B_{17}) \simeq 2.5 \times 10^2 \times \frac{\rho_7 \, Y_{0.5}}{E_{10}} \, ,$$

where $B_{17} = B/(10^{17}\,\mathrm{G})$, $\rho_7 = \rho/(10^7\,\mathrm{g/cm^3})$, $Y_{0.5} = Y_e/0.5$, $E_{10} = E/(10\,\mathrm{MeV})$,

appears to be of the order of $B \gtrsim 10^{18}$ G, far exceeding the maximal magnetic field strength which is believed to arise inside the exploding supernova.

"Neutrino spin light" without photon dispersion in medium

The effect of **"neutrino spin light"** (A. Studenikin et al., 2003-2005) was based on the influence of an active medium on the neutrino dispersion.

An additional Wolfenstein energy acquired by a left-handed neutrino in medium:

$$E_{\nu_L} \simeq E_0 + \frac{G_{\rm F}\, N}{\sqrt{2}} \left(1 + 4\, \sin^2 \theta_{\rm W}\right) \,, \qquad E_{\nu_R} \simeq E_0$$

(N is the number density of background electrons),

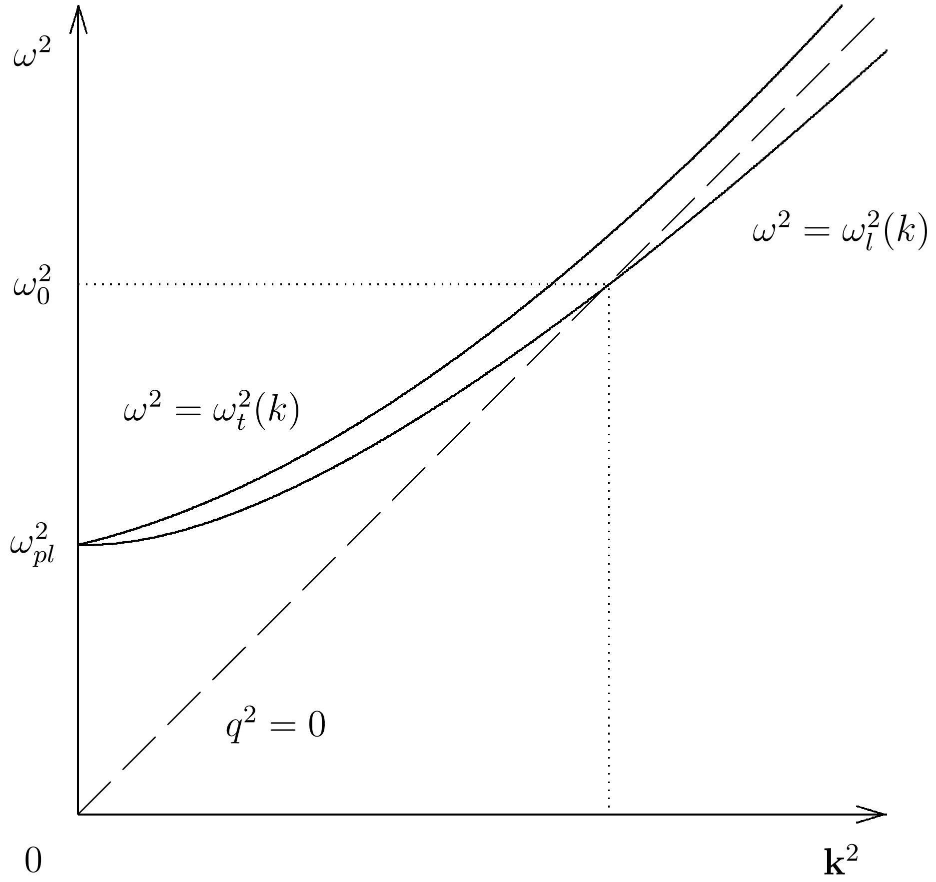
together with the effective $\nu_L \nu_R \gamma$ vertex caused by the neutrino magnetic moment, made possible the decay:

$$\nu_L \to \nu_R + \gamma \,.$$

The kinematical analysis and calculations were made, considering the photon vacuum dispersion, $\omega = |\mathbf{k}|$.

"Neutrino spin light" without photon dispersion in medium?

However, medium modifies the photon dispersion (H. Weldon, 1982; E. Bratten, 1991, etc.) to the form $\omega = |\mathbf{k}|/n$ ($n \neq 1$ is the refractive index). In astrophysical plasma the photon becomes a **plasmon** (transversal t and longitudinal ℓ):



"Neutrino spin light" without photon dispersion in medium?

The scale of deviation of the photon dispersion in matter from the vacuum one, the plasmon frequency:

$$\omega_{\rm pl} = \left(\frac{4\,\alpha}{3\,\pi}\right)^{1/2} \left(3\,\pi^2\,N\right)^{1/3} \simeq 0.73 \times 10^7\,{\rm eV} \left(\frac{N}{10^{37}\,{\rm cm}^{-3}}\right)^{1/3}.$$

The scale of the electron number density N is taken, typical for the interior of a neutron star.

The Wolfenstein energy defining the neutrino dispersion in medium:

$$\Delta E_{\rm W} = \frac{G_{\rm F}\,N}{\sqrt{2}} \left(1 + 4\,\sin^2\theta_{\rm W}\right) \simeq 1.2\,{\rm eV} \left(\frac{N}{10^{37}\,{\rm cm}^{-3}}\right).$$

No "neutrino spin light" because of photon dispersion in medium

The 4-momentum of the transversal plasmon is always timelike, $\omega^2 > \mathbf{k}^2 \, (n < 1)$, and its effective "mass" is much greater than the energy benefit caused by the neutrino dispersion $\Rightarrow$ the decay $\nu_L \to \nu_R \gamma_t$ is kinematically forbidden.

The same for the decay $\nu_L \to \nu_R \gamma_l$ where the 4-momentum of γ_l is timelike.

In the region where the 4-momentum of γ_l is spacelike, $\omega^2 < \mathbf{k}^2 \, (n > 1)$, the decay $\nu_L \to \nu_R \gamma_l$ is kinematically allowed due to the photon dispersion (the neutrino Cherenkov process).

The longitudinal plasmon is unstable here (Landau damping), and the neutrino energy is transformed not into the "light" radiation, but in fact into the energy of excitation of plasma electrons. Thus, the effect of "neutrino spin light" has no physical region of realization.

Conclusions

- We have calculated the neutrino self-energy operator $\Sigma(p)$ in the presence of a magnetic field. Our results strongly disagree with those by E. Elizalde e.a., 2002; 2004. We confirm the result by J. C. D'Olivo e.a., 1989; P. Elmfors e.a., 1996, that the pure magnetic field contribution into the neutrino energy does not exceed the plasma contribution.
- The magnetic field strength needed for solving the problem of the supernova shock wave revival via the possible field-induced resonance enhancement of the neutrino oscillations, far exceeds the maximal magnetic field strength which is believed to arise inside the exploding supernova.
- The effect of "neutrino spin light" has no physical region of realization because of the photon dispersion in medium.