The process $\nu_L \longrightarrow \nu_R + \gamma^*$ in the supernova core conditions and the neutrino magnetic moment

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Publications on the subject

The main results are presented in the paper:

- A new bound on the Dirac neutrino magnetic moment from the plasma induced neutrino chirality flip in a supernova, *Journal of Cosmology and Astroparticle Physics, 2007, in press;* `arXiv:0709.0110[hep-ph]`

see also:

- Plasma induced neutrino radiative decay instead of neutrino spin light, *Modern Physics Letters A. 2006. V. 21. No. 23. P. 1769-1775;* `arXiv:hep-ph/0606262`
- Plasma induced fermion spin-flip conversion $f_L \to f_R + \gamma$, *International Journal of Modern Physics A, 2007, V. 22, No. 19, pp. 3211-3227;* `arXiv:hep-ph/0701228`

Outline

Neutrino spin-flip in the supernova core

Neutrino magnetic moment ⇒ spin-flipping processes
in the supernova core:

$$\nu_L \longrightarrow \nu_R$$

ν_R's being sterile fly away from the core ⇒ *leaving no enough energy to explain the observed luminosity of the supernova* ⇒ upper bound on the neutrino magnetic moment.

Neutrino spin-flip in the supernova core

SN1987A, *R. Barbieri and R. N. Mohapatra (1988)*: the neutrino spin-flip via both $\nu_L e^- \to \nu_R e^-$ and $\nu_L p \to \nu_R p$ scattering processes.

From the ν_R luminosity upper limit $Q_{\nu_R} < 10^{53}$ erg/s, the upper bound on the neutrino magnetic moment was established :

$$\mu_\nu < (0.2 - 0.8) \times 10^{-11} \, \mu_{\rm B} \, .$$

However, the essential plasma polarization effects in the photon propagator were not considered comprehensively. An *ad hoc* photon thermal mass was inserted instead.

Neutrino spin-flip in the supernova core

Later on, *A. Ayala, J. C. D’Olivo and M. Torres (1999)* used the formalism of the **Thermal Field Theory** to take into account the influence of hot dense astrophysical plasma on the photon propagator.

The upper bound for the neutrino magnetic moment was improved by them in the factor of 2:

$$\mu_\nu < (0.1 - 0.4) \times 10^{-11}\, \mu_{\rm B} \, .$$

Neutrino spin-flip in the supernova core

However, looking at the intermediate analytical results by the authors, we conclude that only the contribution of plasma **electrons** was taken into account there, while the **proton** fraction was omitted.

Thus, the reason exists to reconsider the neutrino spin-flip processes in the supernova core more attentively.

We confirm in part, that the neutrino scattering on plasma **protons** is essential, as well as the scattering on plasma **electrons**.

Neutrino spin-flip in the supernova core

The Lagrangian of the interaction of a neutrino with a magnetic moment μ_ν with photons is:

$$\mathcal{L} = -\frac{\mathrm{i}}{2}\,\mu_\nu\,(\bar{\nu}\sigma_{\alpha\beta}\nu)\,F^{\alpha\beta}\,,$$

where $\sigma_{\alpha\beta} = (1/2)\,(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)$, and $F^{\alpha\beta}$ is the tensor of the photon electromagnetic field.

In the supernova core conditions, a plasma influence on the photon dispersion properties must be taken into account.

Neutrino spin-flip in the supernova core

The eigenvalues of the photon polarization tensor $\Pi_{\alpha\beta}$, the functions $\Pi_{(\lambda)}$, define the photon dispersion law:

$$\omega^2 - k^2 - \Pi_{(\lambda)}(\omega, k) = 0 \, ,$$

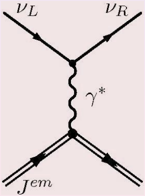
where $\lambda = t, \ell$ mean transversal and longitudinal photon polarizations.

In general, the functions $\Pi_{(\lambda)}$ have imaginary parts. This means, that the "photon" is unstable in plasma, and can not be treated as a real photon.

It would be more self-consistent to consider the neutrino scattering off plasma particles via the intermediate virtual plasmon γ^*.

Neutrino interaction with background

The neutrino chirality flip process of the neutrino scattering via the intermediate virtual plasmon γ^* on the plasma electromagnetic current presented by electrons, $\nu_L e^- \to \nu_R e^-$, protons, $\nu_L p \to \nu_R p$, etc., is shown in the diagram:



Here, J^{em} is an electromagnetic current in the general sense, formed by different components of the medium, i.e. free electrons and positrons, free ions, neutral atoms, etc.

Neutrino interaction with background

The most useful value is the rate $\Gamma_{\nu_R}(E')$ of creation of the right-handed neutrino with the fixed energy E' by all the left-handed neutrinos, being obtained by integration over the states of particles forming the electromagnetic current and over the states of the initial left-handed neutrinos.

Given $\Gamma_{\nu_R}(E')$, one can calculate both the right-handed neutrino flux and the right-handed neutrino luminosity.

The technics of calculations is rather standard. The only principal point is to use the photon propagator $G^{\alpha\beta}(q)$ with taking account of the plasma polarization effects.

Neutrino interaction with background

We take the photon propagator in the form:

$$G^{\alpha\beta}(q) = \frac{\mathrm{i}\, \varrho^{\alpha\beta}_{(t)}}{q^2 - \Pi_{(t)}} + \frac{\mathrm{i}\, \varrho^{\alpha\beta}_{(\ell)}}{q^2 - \Pi_{(\ell)}} \,,$$

where $\varrho^{\alpha\beta}_{(t,\ell)}$ are the density matrices for the transversal and longitudinal photon polarizations,

$$\varrho^{\alpha\beta}_{(t)} = - \left(g^{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2} - \frac{\ell^\alpha \ell^\beta}{\ell^2} \right) , \qquad \varrho^{\alpha\beta}_{(\ell)} = - \frac{\ell^\alpha \ell^\beta}{\ell^2} \,,$$

$\ell_\alpha = q_\alpha \, (u\, q) - u_\alpha \, q^2$, u_α is the 4-vector of the plasma velocity.

The propagator has no ambiguity when the functions $\Pi_{(t,\ell)}$ are real.
In the case of complex functions we use the same form of the propagator with the retarded functions $\Pi_{(t,\ell)}$.

Neutrino interaction with background

There is also such a subtle effect as the additional energy W acquired by a left-handed neutrino in plasma. With this effect, the general expression for the rate of creation of the right-handed neutrino is:

$$\Gamma_{\nu_R}(E') = \frac{\mu_\nu^2}{16\,\pi^2\,E'^2} \int\limits_D \frac{\mathrm{d}q_0\,\mathrm{d}k}{k}\, f_\nu(E'+q_0)\,[1+f_\gamma(q_0)]\,(2E'+q_0)^2\,q^4$$

$$\times \left\{ \left(1 - \frac{k^2}{(2E'+q_0)^2}\right) \left[1 - \frac{2q_0\,W}{q^2} + \frac{8E'(E'+q_0)\,W^2}{q^4\,[(2E'+q_0)^2/k^2 - 1]}\right] \times \right.$$

$$\left. \times \rho_{(t)}(q_0,k) - \left(1 - \frac{2q_0\,W}{q^2}\right) \rho_{(\ell)}(q_0,k) \right\},$$

where $q^2 = q_0^2 - k^2$.

Neutrino interaction with background

$f_\nu(E' + q_0)$ and $f_\gamma(q_0)$ are the neutrino and photon distribution functions, and $\rho_{(\lambda)}$ are the photon spectral density functions:

$$\rho_{(\lambda)} = \frac{2\left(-\mathrm{Im}\,\Pi_{(\lambda)}\right)}{\left(q^2 - \mathrm{Re}\,\Pi_{(\lambda)}\right)^2 + \left(\mathrm{Im}\,\Pi_{(\lambda)}\right)^2}.$$

Our result is in agreement with the rate obtained by *P. Elmfors et al. (1997)*.

However, our result for *the electron contribution* is larger by the factor of 2 than in the papers by *A. Ayala et al.* We believe that an error was made there.

Neutrino interaction with background

Our formula having the most general form can be used for neutrino-photon processes ($\nu_L \to \nu_R \gamma^*$) in any optically active medium. One only needs to specify the photon spectral density functions $\rho_{(\lambda)}$.

For example, in the medium where $\mathrm{Im}\, \Pi_{(t)} \to 0$ in the space-like region $q^2 < 0$ corresponding to the refractive index values $n > 1$, the spectral density function is transformed to δ-function, and the result can be reproduced of the paper by *W. Grimus and H. Neufeld (1993)* devoted to the study of the Cherenkov radiation of **transversal** photons by neutrinos.

Neutrino interaction with background

If one **formally** takes the limit $\mathrm{Im}\,\Pi_{(\ell)} \to 0$, the result obtained by *S. Mohanty and S. Sahu (1997)* can be reproduced, namely, the width of the Cherenkov radiation and absorption of **longitudinal** photons by neutrinos in the space-like region $q^2 < 0$.

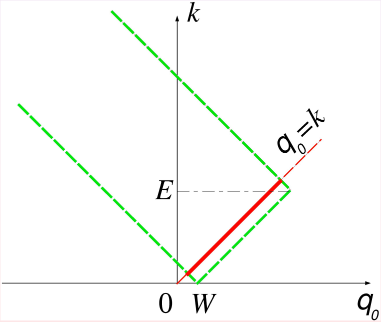
However, the limit $\mathrm{Im}\,\Pi_{(\ell)} \to 0$ itself is unphysical in the real astrophysical plasma conditions considered by those authors and leads to the strong overestimation of a result.

“Neutrino spin light”

One more unphysical case, the so-called “neutrino spin light”, was considered in the series of papers by *A. Studenikin et al. (2003-2006)*, where the photon dispersion in medium was ignored.

“Neutrino spin light”

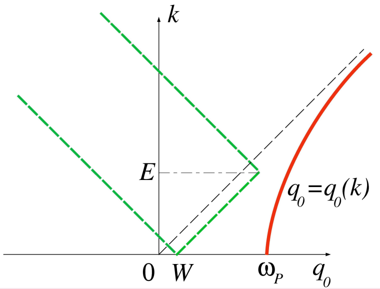
The region of integration for the width $\Gamma^{\text{tot}}_{\nu_L \to \nu_R}$ with the *fixed initial neutrino energy E* would contain (if a photon did not feel plasma) the vacuum dispersion line $q_0 = k$ (the **red bold** line).



“Neutrino spin light”

However, the photon dispersion in plasma is not the vacuum one!

“Neutrino spin light”



For the fixed plasma parameters the threshold neutrino energy $E_{\min}$ exists for the process $\nu_L \to \nu_R \gamma^*$ to be possible.

It is useful to compare the numerical values in the figure.

“Neutrino spin light”

For the interior of a neutron star, the additional energy acquired by a left-handed neutrino in plasma (N_B is the barion density):

$$W \simeq 6\ \mathrm{eV} \left(\frac{N_B}{10^{38}\ \mathrm{cm}^{-3}} \right) ,$$

while the plasmon frequency, defining the photon dispersion:

$$\omega_P \simeq 10^7\ \mathrm{eV} \left(\frac{N_B}{10^{38}\ \mathrm{cm}^{-3}} \right)^{1/3} .$$

“Neutrino spin light”

The threshold neutrino energy in this case:

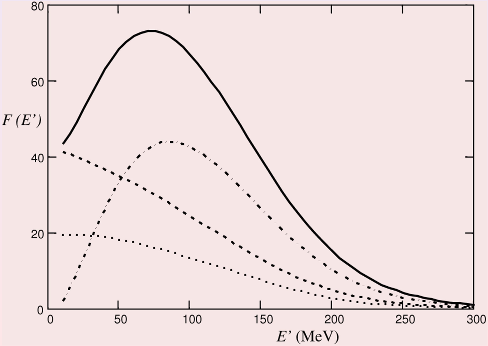
$$E_{\rm min} \simeq \frac{\omega_P^2}{2\,W} \simeq 10\,{\rm TeV}\,.$$

The details can be found in our papers:

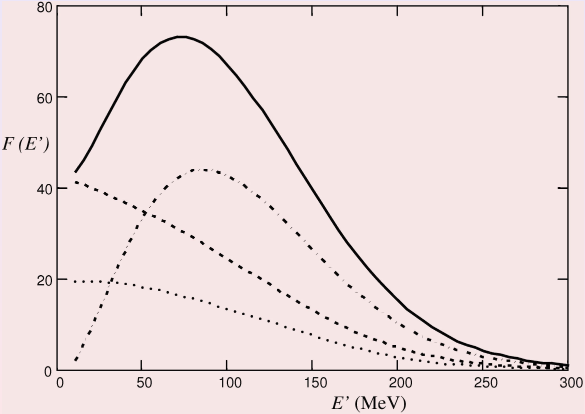
- *Mod. Phys. Lett. A* **21**, *1769 (2006)*, `hep-ph/0606262`;
- *Int. J. Mod. Phys. A* **22**, *3211 (2007)*, `hep-ph/0701228`.

The rate of creation of the right-handed neutrino

The production rate of ν_R: the electron contribution (dashed line), the proton contribution (dash-dotted line), the total rate (solid line) for T =30 MeV. The dotted line shows the result by *A. Ayala et al.*



The rate of creation of the right-handed neutrino



The function $F(E')$ is defined by: $\Gamma_{\nu_R}(E') = (\mu_\nu^2 \; T^3/(32\,\pi))\, F(E')$.

Bound on μ_ν from the right-handed neutrino luminosity

The supernova core luminosity for ν_R emission can be computed as

$$Q_{\nu_R} = V \int \frac{\mathrm{d}^3 p'}{(2\pi)^3} \, E' \, \Gamma_{\nu_R}(E') \, ,$$

where V is the plasma volume.

For the same supernova core conditions as in the earlier papers (plasma volume $V \sim 4 \times 10^{18} \mathrm{cm}^3$, temperature range $T = 30 - 60$ MeV, electron chemical potential range $\tilde{\mu}_e = 280 - 307$ MeV, neutrino chemical potential $\tilde{\mu}_\nu = 160$ MeV), we obtain

$$Q_{\nu_R} = \left(\frac{\mu_\nu}{\mu_\mathrm{B}} \right)^2 (0.38 - 2.2) \times 10^{77} \text{ erg/s} \, .$$

Bound on μ_ν from the right-handed neutrino luminosity

$$Q_{\nu_R} = \left(\frac{\mu_\nu}{\mu_{\rm B}}\right)^2 (0.38 - 2.2) \times 10^{77}\ \text{erg/s}\,.$$

Assuming that $Q_{\nu_R} < 10^{53}$ erg/s, we obtain the upper limit on the neutrino magnetic moment:

$$\mu_\nu < (0.7 - 1.5)\ \times 10^{-12}\,\mu_{\rm B}\,.$$

Remind that the result by *A. Ayala et al.* was:

$$\mu_\nu < (1 - 4)\ \times 10^{-12}\,\mu_{\rm B}\,.$$

Conclusions

- We have investigated in detail the neutrino chirality-flip process under the conditions of the supernova core. The plasma polarization effects caused both by electrons and protons were taken into account in the photon propagator. It is shown in part that the contribution of the proton fraction of plasma dominates. The rate $\Gamma_{\nu_R}(E')$ of creation of the right-handed neutrino with the fixed energy E', the energy spectrum, and the luminosity have been calculated.

Conclusions (cont’d)

- From the limit on the supernova core luminosity for ν_R emission, we have obtained the upper bound on the neutrino magnetic moment $\mu_\nu < (0.7 - 1.5) \times 10^{-12} \mu_{\rm B}$.

Conclusions (cont'd)

- From the limit on the supernova core luminosity for ν_R emission, we have obtained the upper bound on the neutrino magnetic moment $\mu_\nu < (0.7 - 1.5) \times 10^{-12}\, \mu_{\rm B}$.
- We have improved the best astrophysical upper bound on the neutrino magnetic moment by the factor of 2.

"Neutrino spin light" at ultra-high neutrino energies?

At **ultra-high** neutrino energies the local limit of the weak interaction does not describe comprehensively the additional neutrino energy in plasma, and the **non-local** weak contribution must be taken into account.

In a general case, this non-local term *identical for both neutrinos and antineutrinos*, is

$$\Delta^{(\mathrm{nloc})} W_i = - \frac{16\, G_{\mathrm{F}}\, E}{3\sqrt{2}} \left[\frac{< E_{\nu_i} >}{m_Z^2} \left(N_{\nu_i} + \bar{N}_{\nu_i} \right) + \delta_{ie} \frac{< E_e >}{m_W^2} \left(N_e + \bar{N}_e \right) \right] .$$

E is the energy of a neutrino with the flavor i, propagating through plasma, $< E_{\nu_i} >$ and $< E_e >$ are the averaged energies of plasma neutrinos and electrons.

"Neutrino spin light" at ultra-high neutrino energies?

This non-local term is always negative.

Thus, there arises the window (if exists) in the neutrino energies for the process to be kinematically opened, $E_{\min} < E < E_{\max}$.

For example, in the solar interior there is no window for the process with electron neutrinos at all.

Kinematical equivalence of "neutrino spin light" and $\bar{\nu}_e + e^- \longrightarrow \tau^- + \bar{\nu}_\tau$

Let us compare the processes:

$$\nu_L \to \nu_R + \gamma \qquad \bar{\nu}_e + e^- \longrightarrow \tau^- + \bar{\nu}_\tau$$

The energy and momentum conservation in the lab frame:

$$E + W = E' + \omega \qquad E + m_e = E' + \omega$$

$$\mathbf{p} = \mathbf{p}' + \mathbf{k} \qquad \mathbf{p} = \mathbf{p}' + \mathbf{k}$$

The Mandelstam S variable in the lab frame:

$$S = 2\,W\,E + W^2 \qquad S = 2\,m_e\,E + m_e^2$$

Kinematical equivalence of "neutrino spin light" and $\bar{\nu}_e + e^- \longrightarrow \tau^- + \bar{\nu}_\tau$

The Mandelstam S variable in the center-of-mass frame:

$$S = \left(\sqrt{m_\gamma^2 + p'^2} + p'\right)^2 \geqslant m_\gamma^2 \qquad S = \left(\sqrt{m_\tau^2 + p'^2} + p'\right)^2 \geqslant m_\tau^2$$

The threshold value for the initial neutrino energy:

$$E \geqslant E_0 = \frac{m_\gamma^2 - W^2}{2\,W} \simeq \frac{m_\gamma^2}{2\,W} \qquad E \geqslant E_0 = \frac{m_\tau^2 - m_e^2}{2\,m_e} \simeq \frac{m_\tau^2}{2\,m_e}$$

"Neutrino spin light" has a famous precursor?

Why the radiation of a relativistic charged particle in an external magnetic field, termed "spin light" does exist, while the "neutrino spin light" does not ?

Because the influence of a **weak magnetic field** and of **dense matter** on the photon dispersion is rather different.

In **dense matter** giving an additional energy to the left-handed neutrino, a photon acquires the effective mass, while in a **laboratory magnetic field** where the "spin light" was investigated, the photon effective mass is negligibly small.