Neutrino-Photon Processes in Strong Magnetic Field

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Abstract

Neutrino-photon processes $\nu \to \nu\gamma$ and $\gamma \to \nu\bar{\nu}$ are investigated in a strong magnetic field in the framework of the standard model. Possible astrophysical manifestations of the processes considered are analyzed.

1 Introduction

Nowadays, it is generally recognized that astrophysical objects and processes inside them give us unique possibilities [1] for investigations of particle properties under extreme conditions of a high density and/or temperature of matter, and also of a strong magnetic field. A concept of the astrophysically strong magnetic field has been changed in the recent years and now the field is considered as the strong one if it is much greater than the known Schwinger value, $B \gg B_e, B_e = m_e^2/e \simeq 4.41 \cdot 10^{13} G$. Possible mechanisms are now discussed of a generation of such strong fields ($B \sim 10^{15} - 10^{17} G$) in astrophysical cataclysms like a supernova explosion or a coalescence of neutron stars [2, 3].

A strong magnetic field, like a medium, makes an active influence on particle properties. First, it opens new channels forbidden in vacuum, for example, $\gamma \to e^+e^-$ [4], $\gamma \to \gamma\gamma$ [5]. Second, the field can catalyze essentially the decay processes as it takes with the massive neutrino radiative decay $\nu \to \nu'\gamma$ [6]. It was shown [6] that the field-induced amplitude was not suppressed by smallness of neutrino masses and did not vanish even in the case of massless neutrino as opposed to the vacuum amplitude. The decay probability of the neutrino with energy $E_\nu < 2m_e$ was calculated under the assumption that the dispersion relation of the final photon was close to the vacuum one $q^2 = 0$. However the photon dispersion in a strong magnetic field differs significantly from the vacuum dispersion with increasing photon energy [5, 7, 8], so the real photon 4-momentum can appear as a space-like and sufficiently large one ($|q^2| \gg m_\nu^2$). In this case the phase space for the neutrino transition $\nu \to \nu'\gamma$ with $m < m'$ is opened also. It means that the decay probability of ultrarelativistic neutrino $\nu \to \nu'\gamma$ becomes insensitive to the neutrino mass spectrum due to the photon dispersion relation in a strong magnetic field. This phenomenon results in a strong suppression ($\sim m_\nu^2/E_\nu^2$) of the neutrino transition with flavor violation and the diagonal process $\nu_l \to \nu_l\gamma$ ($l = e, \mu, \tau$) is realized only. It means that in a strong magnetic field a question of neutrino mixing is not relevant in the case of the ultrarelativistic $\nu \to \nu\gamma$ transitions which can be considered in the frame of the standard model without lepton mixing.

The photon splitting into the neutrino-antineutrino pair $\gamma \to \nu\bar{\nu}$ is the other interesting manifestation of neutrino processes in the magnetic field. This process well studied in plasma [1, 9] becomes possible in the external field when the photon acquires the timelike 4-momentum q ($q^2 > 0$).

Here we present some results of the strong magnetic field influence on the diagonal neutrino transition $\nu \to \nu\gamma$ [10] and the photon splitting $\gamma \to \nu\bar{\nu}$ [11], and discuss possible astrophysical consequences. Below we consider a possible situation when from both components of the active medium, a magnetic field and plasma, presented in the most of astrophysical objects, the magnetic component dominates. For example, in a supernova explosion or in a coalescence of neutron stars a region could exist outside the neutrinosphere, of order of several hundred kilometers in size, where plasma is rather rarefied, and the magnetic field of the toroidal type [2] could reach a value of $10^{14} - 10^{16}$ G.

If the momentum transferred is relatively small, $|q^2| \ll M_W^2$, the calculations can be performed using the effective electron–neutrino interaction:

$$\mathcal{L} \;=\; \frac{G_F}{\sqrt{2}}\, j_\alpha \left(\bar{e}\gamma_\alpha(g_V - g_A\gamma_5)e\right). \tag{1}$$

Here $j_\alpha = \bar{\nu}\gamma_\alpha(1-\gamma_5)\nu$ is the left neutrino current, $g_V = \pm\frac{1}{2} + 2\sin^2\theta_W$, $g_A = \pm\frac{1}{2}$, the upper signs correspond to the electron neutrino ($\nu = \nu_e$) when both neutral and charged current interaction takes part in a process. The lower signs correspond to μ and τ neutrinos ($\nu = \nu_\mu, \nu_\tau$), when the neutral current interaction is only presented in the Lagrangian (1).

2 Neutrino Bremsstrahlung $\nu \to \nu\gamma$

Here we discuss the high energy diagonal neutrino transition $\nu \to \nu\gamma$ [12, 13, 14, 10] taking into account the photon dispersion in the magnetic field. Note that only two photon eigenmodes with polarization vectors

$$\varepsilon_\mu^{(\|)} = \frac{(q\varphi)_\mu}{\sqrt{q_\perp^2}}, \qquad \varepsilon_\mu^{(\perp)} = \frac{(q\tilde{\varphi})_\mu}{\sqrt{q_\|^2}}, \tag{2}$$

are realized in the magnetic field (the so-called parallel ($\|$) and perpendicular ($\perp$) polarizations according to Adler's notations [5]). Here $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}$ are the dimensionless tensor and the dual tensor of the external magnetic field with the strength $\vec{B}$; q_μ is the photon 4-momentum; $q_\|^2 = (q\tilde{\varphi}\tilde{\varphi}q) = q_\alpha\tilde{\varphi}_{\alpha\beta}\tilde{\varphi}_{\beta\mu}q_\mu$, $q_\perp^2 = (q\varphi\varphi q)$.

The partial amplitudes of the photon creation with the polarizations (2) can be presented in the form:

$$M_\| \;\simeq\; -\frac{eG_F}{\sqrt{2}}\, g_V\, V_\| \,\frac{(q\varphi j)}{\sqrt{q_\perp^2}} \tag{3}$$

$$M_\perp \;\simeq\; -\frac{eG_F}{\sqrt{2}}\, \frac{1}{\sqrt{q_\|^2}} \left\{ g_V\, V_\perp(q\tilde{\varphi}j) + g_A\, A(q\varphi\varphi j)\right\}, \tag{4}$$

where $M_\|$ and $M_\perp$ correspond to the creation of the $\|$ and $\perp$ photon modes, respectively. In general, the functions V_λ and A have cumbersome forms. Here we discuss their properties which are very important for the analysis of this process. Namely, these functions are singular with respect to the variable $q_{\|}^2$ in the threshold points $\mathcal{E}_{nn'}^2 = \left(\sqrt{m_e^2 + 2eBn} + \sqrt{m_e^2 + 2eBn'}\right)^2$. The square-root singularities of the functions V_λ in the threshold points are known as the cyclotronic resonances in the vacuum polarization [7].

It is of interest for some astrophysical applications to investigate the case of relatively high neutrino energy $E_\nu \simeq 10 \div 20\,\mathrm{MeV} \gg m_e$ and strong magnetic field $eB > E_\nu^2$ when a region of the cyclotronic resonance on the ground Landau level dominates in this process. As the analysis shows, the functions $V_\perp$ and A have the square-root singularity in this region:

$$A \simeq -V_\perp \simeq \frac{eB\, m_e\, e^{-q_\perp^2/2eB}}{2\pi\sqrt{4m_e^2 - q_{\|}^2}}. \tag{5}$$

It means that only the amplitude $M_\perp$ (4) which corresponds to the creation of $\perp$ photon mode has the the enhancement due to the square-root singularity on the ground Landau level $q_{\|}^2 \to 4m_e^2$. As for $V_{\|}$, it has not such an enhancement and the dispersion relation for $\|$ photon mode is close to the vacuum one. So, in this region the amplitude $M_{\|} \sim (q\varphi j) \sim q^2$ is small due to the collinear kinematics of the process with creation of $\|$ photon mode. We stress that the vacuum polarization $\mathcal{P}_\perp$ has also the singular behavior in the vicinity of the threshold point $q_{\|}^2 = 4m_e^2$. It means that many-loops quantum corrections are of great importance in the vicinity of the resonance. These radiative corrections are reduced to the renormalization of the wave function of $\perp$ photon mode:

$$\varepsilon_\mu^{(\perp)} \to \sqrt{Z}\,\varepsilon_\mu^{(\perp)}, \quad Z = \left(1 - \frac{\partial \mathcal{P}_\perp}{\partial q_{\|}^2}\right)^{-1}, \tag{6}$$

and lead to the extra factor $\sqrt{Z}$ in the amplitude $M_\perp$ (4). The main contribution to the probability of $\nu \to \nu\gamma^{(\perp)}$ is determined from the vicinity of the resonance point $q_{\|}^2 = 4m_e^2$ and in the limit $eB \gg E_\nu^2 \sin^2\theta$ has the form:

$$W^{(\gamma)} \simeq \frac{\alpha G_F^2}{8\pi^2}\,(g_V^2 + g_A^2)\,(eB)^2\,E_\nu\,\sin^2\theta. \tag{7}$$

Here E_ν is the initial neutrino energy, θ is the angle between the vectors of the magnetic field strength $\vec{B}$ and the momentum of the initial neutrino $\vec{p}$.

3 The Neutrino Energy and Momentum Losses

It should be noted that a practical significance of these processes for astrophysics could be in the mean values of the neutrino energy and momentum losses rather than in the probabilities. These mean values could be found from the four-vector

$$Q^\alpha = E_\nu \int dW \; q^\alpha = \frac{1}{16\pi^3} \int \frac{d^3k}{\omega} \, q^\alpha \, Im \, \mathcal{M}(\nu\bar{\nu} \to \nu\bar{\nu}). \tag{8}$$

Its zero component is connected with the mean neutrino energy loss in a unit time, $\mathcal{I} = dE/dt$. The space components of the four-vector (8) are connected similarly with the neutrino momentum loss in unit time, $\vec{\mathcal{F}} = d\vec{p}/dt$. Here we present the expression for Q^α in the limiting case when $eB \gg E^2 \sin^2\theta$:

$$\mathcal{I} = \frac{1}{4} EW \left(1 + \frac{2 g_V g_A}{g_V^2 + g_A^2} \cos\theta \right), \tag{9}$$

$$\mathcal{F}_z = \frac{1}{4} EW \left(\cos\theta + \frac{2 g_V g_A}{g_V^2 + g_A^2} \right), \quad \mathcal{F}_\perp = \frac{1}{2} EW \; \sin\theta, \tag{10}$$

where the z axis is directed along the field, the vector $\vec{\mathcal{F}}_\perp$, transverse to the field, lies in the plane of the vectors $\vec{B}$ and $\vec{p}$.

4 Possible Astrophysical Consequences

To illustrate a possible application of our results, below we estimate the neutrino energy and momentum losses in an astrophysical cataclysm of the type of a supernova explosion or a merger of neutron stars. We assume that for some reasons a compact remnant has a strong poloidal magnetic field of order $B \sim 10^{15} \div 10^{17}$ G. Objects of this type, the so-called “magnetars”, were investigated in [15]. According to the standard astrophysical models [16], the neutrinos of all species with the typical mean energy $\bar{E}_\nu \sim 20$ MeV are radiated from a neutrinosphere in above mentioned astrophysical cataclysm. In this case the neutrino propagating through the magnetic field will lose the energy and the momentum in accordance with our formulae. The main contribution to the total energy lost by neutrinos in the field could be estimated from Eq. (9):

$$\frac{\Delta\mathcal{E}}{\mathcal{E}_{tot}} \simeq 0.6 \cdot 10^{-2} \left(\frac{B}{10^{17}\,\mathrm{G}}\right) \left(\frac{\bar{E}}{20\,\mathrm{MeV}}\right)^3 \left(\frac{\Delta\ell}{10\,\mathrm{km}}\right). \tag{11}$$

Here $\Delta\ell$ is a characteristic size of the region where the field strength varies insignificantly, $\mathcal{E}_{tot}$ is the total energy carried off by neutrinos in a supernova explosion, $\bar{E}$ is the neutrino energy averaged over the neutrino spectrum. Here we take the energy scales which are believed to be typical for supernova explosions [16].

The asymmetry of outgoing neutrinos $A = |\sum_i \mathbf{p}_i|/\sum_i |\mathbf{p}_i|$ is another interesting manifestation. In the same limit of the strong field we obtain

$$A \sim 10^{-2} \left(\frac{B}{10^{17}\,\mathrm{G}}\right) \left(\frac{\bar{E}}{20\,\mathrm{MeV}}\right)^3 \left(\frac{\Delta\ell}{10\,\mathrm{km}}\right). \tag{12}$$

One can see from Eqs. (11) and (12) that the effect could manifest itself at a level of about percent. In principle, it could be essential in a detailed theoretical description of the process of supernova explosion.

We note that γ-quanta produced are captured by the strong magnetic field and propagate along the field. At the first glance, it seems photons are confined. However the magnetosphere of a "magnetar" has a "polar cap" region which is defined as a narrow cone along the magnetic field axis with open lines of the magnetic field strength. So the particles which are created in the mentioned above neutrino reactions within a narrow cone can escape outside. Below we estimate the neutrino energy loss in the "polar cap" regions of a millisecond "magnetar" taking into account that both processes are comparable for the values of the physical parameters:

$$\begin{aligned} \mathcal{E} \;&\sim\; 10^{48}\,\mathrm{erg} \left(\frac{\mathcal{E}_{tot}}{3 \cdot 10^{53}\,\mathrm{erg}}\right) \left(\frac{B}{10^{17}\,\mathrm{G}}\right)^2 \\ &\times\; \left(\frac{\bar{E}}{20\,\mathrm{MeV}}\right) \left(\frac{R}{10\,\mathrm{km}}\right)^3 \left(\frac{10^{-3}\,\mathrm{sec}}{P}\right)^2, \end{aligned} \tag{13}$$

where $\mathcal{E}_{tot} \sim 10^{53}$ erg is the typical total neutrino radiation energy; P is the "magnetar" rotation period [3]; B is the magnetic field strength in the vicinity of the neutrinosphere of radius R.

We point out that the energy loss (13) in terms of 4π-geometry is close to energy deposition observed as γ-ray bursts (GRB's). In the standard "fireball" model energy of order of 10^{51} erg is deposited in a small volume and results in an ultrarelativistic ejecta. A collision of the ejecta with the intergalactic medium can be a source of the GRB [17]. It is interesting that rapid rotation of the remnant combined with the strong magnetic field becomes popular for understanding GRB's afterglow [18].

5 Photon Splitting $\gamma \to \nu\bar{\nu}$

The photon splitting $\gamma \to \nu\bar{\nu}$ is the crossed channel of the gamma radiation by a neutrino $\nu \to \nu\gamma$. Due to the photon dispersion in a magnetic field [19], the necessary condition of the splitting, $q^2 > 0$, is realized for the perpendicular ($\perp$) polarization in the region $q_{\|}^2 > 4m_e^2$ and for the parallel ($\|$) polarization in the region $q_{\|}^2 > (m_e + \sqrt{m_e^2 + 2eB})^2$. However, due to the collinearity of the kinematics, $j_\alpha \sim q_\alpha \sim p_\alpha \sim p'_\alpha$, the amplitude of the $\|$ mode splitting is suppressed.

The total probability of $\gamma^{(\perp)} \to \nu\bar{\nu}$ into all neutrino species is defined by the expression

$$W_\perp = \frac{1}{16\pi\omega} \int_0^1 dx \, |M_\perp|^2 = \frac{\alpha \, G_F^2}{16\pi^4\omega} \, e^2 (qFFq) \, |J|^2. \tag{14}$$

Here we assume that all neutrino masses are much smaller than the field-induced "photon mass". In the case when the field strength B appears to be the largest physical parameter, $eB \gg q_{\|}^2$, the integral J is:

$$J \simeq \frac{1 - v^2}{2v} \left(\ln \frac{1 + v}{1 - v} - i\pi \right) + 1, \tag{15}$$

where $v = \sqrt{1 - 4m_e^2/q_{\|}^2}$.

Let us note that the photon splitting probability (14) with (15) contains at first sight the pole-type singularity at $q_{\|}^2 \to 4m_e^2$ ($v \to 0$). However, the solution of the equation of the photon dispersion in this limit shows that:

$$|q_{\|}^2 - 4m_e^2|_{min} = \omega \, \Gamma_{\gamma \to e^- e^+}. \tag{16}$$

It is known that the similar seeming singularity, but of the square-root-type, takes place in the process of the photon splitting into the electron-positron pair $\gamma \to e^+e^-$ [4]. As was shown in Ref. [19], taking account of the photon dispersion in the process $\gamma \to e^+e^-$ leads to a finite value for the decay width, which is maximal at the point $q_{\|}^2 = 4m_e^2$

$$(\Gamma_{\gamma \to e^+e^-})_{max} = \frac{\sqrt{3}}{2} \left(\frac{2\alpha eB}{m_e^2} \right)^{2/3} \frac{m_e^2}{\omega}. \tag{17}$$

The probability of the splitting $\gamma \to \nu\bar{\nu}$ is also finite, in view of Eqs. (16) and (17), and amounts up to the maximal value:

$$(W_\perp)_{max} = \frac{(G_F\, m_e^2)^2}{4\sqrt{3}\pi^2} \left(\frac{2\alpha eB}{m_e^2} \right)^{1/3} \frac{eB}{\omega}. \tag{18}$$

It is evident that the probability (18) of the electroweak process $\gamma \to \nu\bar{\nu}$ is suppressed by the factor $(G_F\, m_e^2)^2$ as compared with the probability (17) of the pure electromagnetic process $\gamma \to e^+e^-$. However, the process with neutrinos could play a role of an additional channel of stellar energy-loss.

The energy-loss rate per unit volume of the photon gas due to the splitting $\gamma \to \nu\bar{\nu}$ is defined by

$$Q = \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\omega/T} - 1}\, \omega\, W_\perp. \tag{19}$$

Substitution of the probability (14) into Eq. (19) gives

$$Q = \frac{\alpha\, G_F^2}{8\pi^4}\, m_e^5\, (eB)^2\, \mathcal{F}(T) \simeq 0.96 \cdot 10^{18}\, \frac{erg}{cm^3\ s} \left(\frac{B}{B_e} \right)^2 \mathcal{F}(T). \tag{20}$$

The temperature function $\mathcal{F}(T)$ in the strong field limit, $eB \gg T^2 \gg m_e^2$, takes the form

$$\mathcal{F}(T) \simeq \frac{T}{2m_e} \left(\ln \frac{T}{m_e} \ln \frac{4T}{\Gamma_\gamma} - 0.187 \right). \tag{21}$$

Here the main contribution arises from the vicinity of the resonance $q_{\|}^2 \sim 4m_e^2$. The width Γ_γ should be taken from Eq. (17) at $\omega = 2m_e$.

In conclusion, we estimate the contribution of the photon splitting $\gamma \to \nu\bar{\nu}$ into the neutrino luminosity in a supernova explosion, from a region of order of a hundred kilometers in size outside the neutrinosphere, where a rather strong magnetic field of the toroidal type could exist:

$$\frac{dE}{dt} \sim 10^{45}\, \frac{erg}{s} \left(\frac{B}{10^{15} G}\right)^2 \left(\frac{T}{2 MeV}\right)^5 \left(\frac{R}{100 km}\right)^3 . \tag{22}$$

It is obviously much smaller than the total neutrino luminosity from the neutrinosphere $\sim 10^{52} erg/s$. It is interesting, however, that the process $\gamma \to \nu\bar{\nu}$ could give an appreciable contribution, equal for all neutrino species, to the low-energy part of the neutrino spectrum.

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